

Syllabus for BME graduate Biomedical Mathematical Methods (16:125:501)

BME 116 M, W 2:00-3:20

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Grader: TBD

I support Rutgers' commitment to diversity, and welcome students of all races, sexes, genders and gender identities, sexual orientations, ages, backgrounds, citizenships, disabilities, ethnicities, family statuses, languages... All are welcome.

Academic Integrity Policy:

The policy for this course is as follows. Penalties for infractions of this policy are at the discretion of the instructor; typically for the first infraction the instructor will meet with the student and the student will get a zero for that assignment, for the second infraction the Graduate School will be notified, and for the third infraction the student will fail the course. Integrity policies for other courses may differ.

- 1) For homeworks, students are encouraged to work together in teams of no more than 4 students. At the top of each homework submitted, students must state who they collaborated with. Unless otherwise explicitly stated, homework must come from the student teams themselves: solutions may not be copied from any source, written or electronic.
- 2) For projects and for take-home exams, students must work entirely on their own: no collaboration with any other person in the course or out of the course is permitted. Any materials used for the exam must be clearly cited, and no materials from any source may be copied onto the exam.
- 3) For in-class exams, students must work entirely on their own. Specific rules for these exams will be posted in advance.

Office hours:

I am available at two help sessions a week, and individual appointments can be made by email: shinbrotkinetics@gmail.com. Tentatively help sessions will be online (zoom) Thursday 8-10 pm and Sunday 1-3 pm, but this is flexible if these times conflict with student schedules, and in-person sessions are available as needed.

Grading:

Unless otherwise required (e.g. due to pandemic or other unusual conditions), course grades will be based on 30% for 2 exams; 30% for homeworks, and 40% for projects.

Homework & Project policy:

Homeworks will be due at 1 pm on class days, 1 week after they are assigned. We will go through homework solutions in class and in help sessions, so late homeworks will not be accepted.

Homeworks will involve analytic work as well as Mathematica and Matlab assignments.

Two projects will be required: students will get ample notice (at least 2-3 weeks) and details will be provided and discussed in class. Projects are students' opportunity to shine through creative analysis of a BME modeling problem. Problem suggestions will be provided, but I encourage, urge, implore students to come up with their own project ideas. I will need to vet these to ensure that they are achievable and appropriate, but the best way to do well and to get the most out of this course is through creative applications.

Texts – these are optional, but highly recommended:

SH Strogatz, Nonlinear Dynamics and Chaos. Students may use any edition available. *This is the book I recommend for students, especially with strong biology backgrounds, who want to understand the foundations of the work that we will cover.*

DG Zill & WS Wright, Advanced Engineering Mathematics. Students may use any edition available. *This is the book I recommend for students, especially with strong engineering*

backgrounds, who want a future reference including multiple methods, applications, and solved problems.

Another excellent resource is Kurt Bryan's Differential Equations: A Toolbox for Modeling the World, available online at <https://www.simiode.org/>.

Specific references within the first two texts follow for students to read the material in advance or as we cover the corresponding material in class:

Strogatz, Chapter 2 Flow on the Line
Strogatz, Chapter 3 Bifurcations
Strogatz, Chapter 4 Flows on the Circle
Strogatz, Chapter 5 Linear Systems
Strogatz, Chapter 6 Phase Plane
Strogatz, Chapter 7 Limit Cycles
Strogatz, Chapter 8 Bifurcations revisited

Zill & Wright, Chapter 1 Introduction to Differential Equations
Zill & Wright, Chapter 2 First-Order Differential Equations
Zill & Wright, Chapter 6 Numerical Solutions of Ordinary Differential Equations
Zill & Wright, Chapter 8, section 8 The Eigenvalue Problem
Zill & Wright, Chapter 10 Systems of Linear Differential Equations
Zill & Wright, Chapter 11 Systems of Nonlinear Differential Equations
Zill & Wright, Chapter 13 Higher-Order Differential Equations

The chapters listed are from Strogatz, edition 1, and Zill & Wright, edition 4 - likely the same in further editions, but I list the chapter titles in case things have been moved around in later editions.

Software:

The course will make extensive use of both Mathematica and Matlab, which are available free of charge to full time students through <https://software.rutgers.edu>.

Wellness:

This is a difficult time for everyone, and counseling, alcohol/drug assistance and psychiatric services (CAPS) are available and staffed by professionals to help you. There is no shame in seeking help: I have used CAPS, as have my students and colleagues.

You can reach CAPS [here, or](#) phone: (848) 932-7884

Course goal and overview:

The course will provide students with the necessary mathematical tools to address advanced engineering problems using a quantitative formulation. Students will also develop a proficiency in using computer programming (i.e. Matlab) to solve the mathematical formulations of engineering problems. In overview, topics covered will include: First and second order ODEs, Linear Algebra, Fourier Analysis, Complex Analysis, Statistics, Linear and nonlinear PDEs. All work will be example-driven, with a focus on Biomedical examples.

Course plan:

A typical plan from past years follows. There is flexibility depending on student abilities & interests, *so tell me if we're going too fast or too slow or if you want to focus on particular areas.*

Topic:	Examples:
Overview: linearity & nonlinearity	
Population growth:	Logistic model
Spread of disease:	SIR model
	Zombie model
	HIV model <i>if of interest to class</i>
First project assignment during week 2	
Multi-body systems:	Synchronization
	Stockbridge damper
	Cheyne-Stokes breathing <i>if of interest to class</i>
Importance of details:	Gompertz model
Numerical integration:	Euler, RK2, RK4
	Symplectic integrators
Bifurcations:	Transcritical - Allee model
	Saddle-node – autocatalysis model
	Multiple saddle-node – Gene switch model
	Pitchfork – Duffing model, budworm catastrophe
Challenge to intuition:	Freefall vs. Falling chains
Activation potential:	Laser-threshold or neuronal firing models
	Subcritical pitchfork bifurcation
First project due week 7, second project assigned week 8. Midterm possible ~ week 8 (will be skipped if students keep up with work)	
Predator-Prey:	Immunology model
	Covid model <i>if of interest to class</i> ; Effects of delay
Phase space:	Pendulum model
Poincaré surface of section:	Hénon model, intro to chaos
Meaning of eigenvalues:	Detailed analysis of coupled oscillators
	Euler formula
	Robin & Jules examples, RCISS data
Symmetries:	Gaits, pronking, tapped cup models
	Spinning book (tennis racket theorem)
	Deformation tensors
Reduction of order:	Use in RK4
	Stability in 2D: center, spiral, node, saddle points
Nonlinearities:	Jacobian, pendulum phase plot
Bifurcations in NL probs:	Streamplots, node $\leftrightarrow$ saddle examples
	Computations: Manipulate or sliders or parsefiles
	2D stability plots using trace & determinant
Second project due week 12	
Lotka-Volterra systems:	Rabbit-fox
	Rabbit-sheep
	Predator-prey-scavenger
Limit cycles:	Inflammation
(Poincaré-Bendixson)	Immune-pathogen competition
	Autocatalysis
	Van der Pol analysis
	Fitzhugh-Nagumo vs. Hodgkin-Huxley
Nullcline analysis:	Glycolysis (Schnakenberg model)
Cobweb plots:	Intermittency types
	Hopf, period doubling, flip bifurcations
Model building <i>if of interest to class</i> :	Whitney-Takens-Ruelle,
	fractal dimension, meaning & measurement
	Lorenz, Rössler, Möbius strip
	Winding numbers & mixing
FINAL exam (week 14+)	